MATH 280 Discrete Mathematical Structures Assignment #8 Name ____________________

The point values for each question is given within []. The total number of points for this assignment is 40.

[2] 1. Compute the transitive closure of the relation $\{(0,1),(1,2),(2,3)\}$

[5] 2. Let $A=\{1,2,3,4\}$ and $B=\{a,b,c,d\}$. Determine which of the following are functions.

- $f \subseteq A \times B$, where $f=\{(1,a),(2,b),(3,c),(4,d)\}$.
- $g \subseteq A \times B$, where $g=\{(1,a),(2,a),(3,b),(4,d)\}$.
- $h \subseteq A \times B$, where $h=\{(1,a),(2,b),(3,c)\}$.
- $k \subseteq A \times B$, where $k=\{(1,a),(2,b),(2,c),(3,a),(4,a)\}$.
- $p \subseteq A \times A$, where $p=\{(1,a),(1,1),(2,1),(3,1),(4,1)\}$.

[10] 3. For each of the following mappings from the given domain to the given codomain, write **N** if the mapping is **not** a function, write **F** if the mapping is a function that is neither injective nor surjective, write **S** if the mapping is a surjective function, write **I** if the mapping is an injective function, write **B** if the mapping is a bijective function, and, if the mapping is a bijection, provide f^{-1}.

(a) $f:\mathbb{Z} \to \mathbb{N}, \quad f(x)=x^2+1$

(b) $f:\mathbb{Z} \to \mathbb{Q}, \quad f(x)=\dfrac{1}{x}$

(c) $f:\mathbb{Z} \times \mathbb{N} \to \mathbb{Q}, \quad f(x,y)=\dfrac{x}{y+1}$

(d) $f:\mathbb{N} \to \mathbb{N}, \quad f(x)=2^x$

(e) $f:\mathbb{N} \to \mathbb{N}, \quad f(x)=\begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

[4] 4. If A and B are both finite, how many different functions are there from A to B?

[4] 5. Prove that the set $\mathbb{N} \times \mathbb{N}$ is countable.

6. Consider the following recurrence relation:

$$T(1)=1$$
$$T(n)=2T(n-1)+1$$

[2] (a) Write the first five elements of the sequence.

[5] (b) Solve the recurence relation subject to the basis step.

(c) Test your solution to 6b by writing a recursive function modeling the original recurrence relation and a second function modeling your solution. Ensure that the two functions produce the same results given equivalent inputs.

[4]

[4] (d) Realizing that testing has its limitations, prove that your closed-form solution is correct.