Graphics Primitives

Chapter 3

Lines

Simplest strategy:

1. Compute slope: $m = \frac{\Delta y}{\Delta x}$
2. Plot the leftmost pixel: (x_0, y_0)
3. Within a loop:
   - Increment x by 1: $x_{i+1} \leftarrow x_i + 1$
   - Calculate $y_i \leftarrow mx_i + b$, for each x_i
   - Plot pixel at $(x_i, \mathrm{round}(y_i))$
     - $\mathrm{round}(a) = \lfloor a + 0.5 \rfloor$

Problem with Simplest Strategy

Inefficient: Each iteration requires

1. Floating point multiplication
2. Addition
3. Call of **floor** function

Improvements on the Simplest Strategy

Eliminate the multiply

$$\begin{aligned} y_{i+i} &= m \cdot x_{i+1} + b \\ &= m(x_i + \Delta x) + b \\ &= m \cdot x_i + m \cdot \Delta x + b \\ &= m \cdot x_i + b + m \cdot \Delta x \\ &= y_i + m \cdot \Delta x \end{aligned}$$

Improvements on the Simplest Strategy

If $\Delta x = 1$, then $y_{i+1} = y_i + m$

Thus, a unit change of x changes y by m

$$x_{i+1} = x_i + 1 \qquad y_{i+1} = y_i + m$$

This is an *incremental algorithm*:

$$a_{i+1} = f(a_i)$$

Adjustment for Steep Slopes

If $|m| > 1$, then a step in x creates a step in $y > 1$

Reverse the roles of x and y:

$$\begin{aligned} \Delta y &= 1 \\ y_{i+i} &= y_i + 1 \end{aligned}$$

Adjustment for Steep Slopes

$$
\begin{aligned}
x_{i+1} &= x_i + \Delta x \\
&= x_i + \frac{\Delta y}{m} \\
&= x_i + \frac{1}{m}
\end{aligned}
$$

DDA Algorithm

- Digital Differential Analyzer
- A DDA is a mechanical device that solves differential equations by numerical methods
- Traces (x, y) by simultaneously incrementing x and y by small steps proportional to the first derivative of x and y

DDA Algorithm Problems

- Since floating point numbers have limited precision,

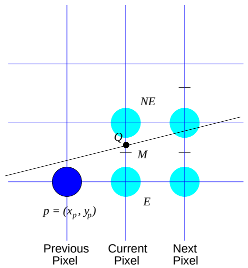
$$\frac{dy}{dx} = m$$

  is inexact
  - Summing an inexact m repetitively leads to cumulative errors that deviate from the true round(y_i)
  - Not a problem for short lines
- The floating point computations take time, round() takes time

Bresenham's Line Drawing Algorithm

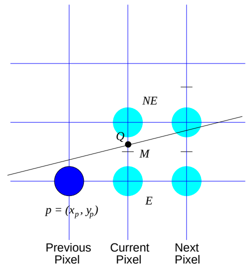
- Also called the Midpoint Line Algorithm
- This development is based on slopes between 0 and 1
  - For other slopes use reflection about an octant axis

Selecting the Next Pixel



- Calculate distance between points Q and NE
- Calculate distance between points Q and E
- Select the closer pixel

Midpoint Perspective



- If the line is above M, choose NE
- Else choose E
- Error is always $\leq \frac{1}{2}$ pixel

Need a way to calculate which side of the line the midpoint lies

Explicit Function

Represent the line as an *implicit function*:

$$y = mx + b \longrightarrow Ax + By + C = 0$$

$$f(x,y) = Ax + By + C = 0$$

(Note: B is not the b of slope-intercept form)

Explicit Function

If $\Delta x = x_1 - x_0$ and $\Delta y = y_1 - y_0$, then the slope-intercept form is

$$y = \frac{\Delta y}{\Delta x} x + b$$

Thus

$$0 = x\Delta y - y\Delta x + b\Delta x = f(x,y)$$

and

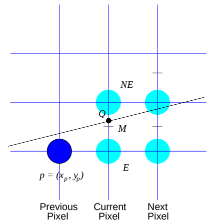
$$A = \Delta y, \;\; B = -\Delta x, \;\; C = b\Delta x$$

$$f(x,y) \begin{cases} > 0 & \text{if } (x,y) \text{ is \textbf{above} the line} \\ = 0 & \text{if } (x,y) \text{ is \textbf{on} the line} \\ < 0 & \text{if } (x,y) \text{ is \textbf{below} the line} \end{cases}$$

Decision Variable

For the midpoint, $f(M) = f(x_p + 1, y_p + \frac{1}{2})$

Test the sign of $f(M)$



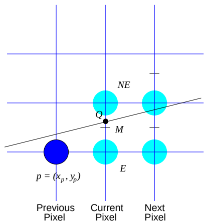
Let d be a *decision variable*:

$$\begin{aligned} d &= f(x_p + 1, y_p + \tfrac{1}{2}) \\ &= A(x_p + 1) + B(y_p + \tfrac{1}{2}) + C \end{aligned}$$

$$d \begin{cases} > 0 & \text{choose } NE \\ = 0 & \text{choose either } (E) \\ < 0 & \text{choose } E \end{cases}$$

E Chosen

Increment M in x direction if E is chosen



$$\begin{aligned} d_{\text{new}} &= f(x_p + 2, y_p + \tfrac{1}{2}) \\ &= A(x_p + 2) + B(y_p + \tfrac{1}{2}) + C \end{aligned}$$

$$d_{\text{old}} = A(x_p + 1) + B(y_p + \tfrac{1}{2}) + C$$

$$d_{\text{new}} - d_{\text{old}} = A$$

E Chosen (cont.)

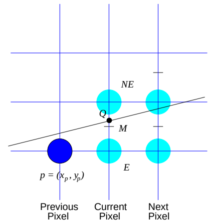
Therefore

$$\begin{aligned} d_{\text{new}} - d_{\text{old}} &= A \\ d_{\text{new}} &= d_{\text{old}} + A \\ \Delta_E &= A \\ &= \Delta y \end{aligned}$$

Thus $f(M)$ need not be computed each time; Δ_E need only be added

NE Chosen

Increment M in both x and y directions if NE is chosen



$$\begin{aligned} d_{\text{new}} &= f(x_p + 2, y_p + \tfrac{3}{2}) \\ &= A(x_p + 2) + B(y_p + \tfrac{3}{2}) + C \end{aligned}$$

$$d_{\text{old}} = A(x_p + 1) + B(y_p + \tfrac{1}{2}) + C$$

$$d_{\text{new}} - d_{\text{old}} = A + B$$

NE Chosen (cont.)

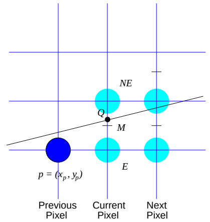
Therefore

$$\begin{aligned} d_{\text{new}} - d_{\text{old}} &= A + B \\ d_{\text{new}} &= d_{\text{old}} + A + B \\ \Delta_{NE} &= A + B \\ &= \Delta y - \Delta x \end{aligned}$$

Thus, d (the decision variable) is incremented by Δ_E or Δ_{NE} depending on which next pixel is chosen.

Initial Values

First pixel is (x_0, y_0)



Initial d is:

$$\begin{aligned} M &= (x_0 + 1, y_0 + \tfrac{1}{2}) \\ f(M) &= A(x_0 + 1) + B(y_0 + \tfrac{1}{2}) + C \\ &= Ax_0 + By_0 + C + A + \frac{B}{2} \\ &= f(x_0, y_0) + A + \frac{B}{2} \end{aligned}$$

Since (x_0, y_0) is on the line, $f(x_0, y_0) = 0$

Initial Values (cont.)

Since (x_0, y_0) is on the line, $f(x_0, y_0) = 0$

Thus,

$$\begin{aligned} d_{\text{start}} &= A + \frac{B}{2} \\ &= \Delta y - \frac{\Delta x}{2} \end{aligned}$$

To eliminate the fraction, let $f(x, y) = 2(Ax + By + C)$

This multiplies the decision variable by 2, **not affecting its sign**