

Parametric Bicubic Surfaces

Parametric Bicubic Surfaces

- Generalization of parametric cubic curves
 - Hermite, Bezier, B-spline
- There are 3D surface equivalents for each of these 3D curve types

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Parametric Cubic Curves

- Recall the general form:

$$Q(t) = G \cdot M \cdot T$$

- G is the geometry matrix representing the position of the control points

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Generalize to a Surface

- Replace parameter t with s :

$$Q(s) = G \cdot M \cdot S$$

- Allow the four control points to vary in 3D along a path parameterized on t :

$$Q(s, t) = [G_1(t) \ G_2(t) \ G_3(t) \ G_4(t)] \cdot M \cdot S$$

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Fix Parameter t

$$Q(s, t) = [G_1(t) \ G_2(t) \ G_3(t) \ G_4(t)] \cdot M \cdot S$$

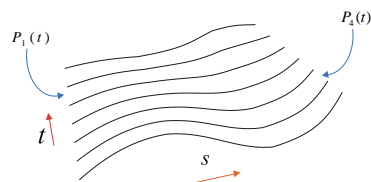
- For a fixed parameter t_1 , $Q(s, t_1)$ is a curve since each $G_i(t_1)$ is constant
- Let t to assume a new value, t_2 , where $t_2 - t_1$ is small, $Q(s, t_2)$ is a slightly different curve
- Repeat this process for a number of values for t between 0 and 1

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The Result?

- The family of curves define a surface



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Surface Patches

- In the case of B-spline curves, four control points at a time controlled segments of the curve
- These segments were smoothly spliced together
- Similarly, 16 control points define a B-spline surface *patch* that smoothly integrates with neighboring patches

Example

```

-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 1 0 -2 1 0 -2 1 0 -1 1 2 0 0 1 3 1 1 2 0 2 1 0 2 1 0 2 1 0
-2 0 0 -2 0 0 -2 0 0 -1 0 3 0 0 5 1 0 3 2 0 0 2 0 0 2 0 0
-2 -1 0 -2 -1 0 -2 -1 0 -1 1 2 0 0 1 3 1 1 2 0 2 -1 0 2 -1 0 2 -1 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0

```

A 9 × 9 grid of control points that define a B-spline surface

Example

```

-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 1 0 -2 1 0 -2 1 0 -1 1 2 0 0 1 3 1 1 2 0 2 1 0 2 1 0 2 1 0
-2 0 0 -2 0 0 -2 0 0 -1 0 3 0 0 5 1 0 3 2 0 0 2 0 0 2 0 0
-2 -1 0 -2 -1 0 -2 -1 0 -1 1 2 0 0 1 3 1 1 2 0 2 -1 0 2 -1 0 2 -1 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0

```

Example

```

-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 1 0 -2 1 0 -2 1 0 -1 1 2 0 0 1 3 1 1 2 0 2 1 0 2 1 0 2 1 0
-2 0 0 -2 0 0 -2 0 0 -1 0 3 0 0 5 1 0 3 2 0 0 2 0 0 2 0 0
-2 -1 0 -2 -1 0 -2 -1 0 -1 1 2 0 0 1 3 1 1 2 0 2 -1 0 2 -1 0 2 -1 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0

```

Example

```

-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 1 0 -2 1 0 -2 1 0 -1 1 2 0 0 1 3 1 1 2 0 2 1 0 2 1 0 2 1 0
-2 0 0 -2 0 0 -2 0 0 -1 0 3 0 0 5 1 0 3 2 0 0 2 0 0 2 0 0
-2 -1 0 -2 -1 0 -2 -1 0 -1 1 2 0 0 1 3 1 1 2 0 2 -1 0 2 -1 0 2 -1 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0

```

Example

```

-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 2 0 -2 2 0 -2 2 0 -1 2 0 0 2 0 1 2 0 2 2 0 2 2 0 2 2 0
-2 1 0 -2 1 0 -2 1 0 -1 1 2 0 0 1 3 1 1 2 0 2 1 0 2 1 0 2 1 0
-2 0 0 -2 0 0 -2 0 0 -1 0 3 0 0 5 1 0 3 2 0 0 2 0 0 2 0 0
-2 -1 0 -2 -1 0 -2 -1 0 -1 1 2 0 0 1 3 1 1 2 0 2 -1 0 2 -1 0 2 -1 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0
-2 -2 0 -2 -2 0 -2 -2 0 -1 -2 0 0 -2 0 1 -2 0 2 -2 0 2 -2 0 2 -2 0

```