MATH 280 Discrete Mathematical Structures Assignment #3 Name ________________

The point values for each question is given within []. The total number of points for this assignment is 37.

Most of these problems have a single number for an answer. For full credit (or partial credit if your answer is incorrect), show how you obtained your result.

[4] 1. Given the truth values A **true**, B **false**, C **true**, what is the truth value of each of the following statements?

(a) $A \wedge (B \vee C)$
(b) $(A \wedge B) \vee C$
(c) $\neg(A \vee B) \wedge C$
(d) $\neg A \vee (\neg B \wedge C)$

[8] 2. Construct truth tables for the following statements. Show intermediate results in extra columns. Note any tautologies or contradictions.

(a) $A \wedge (\neg A \vee \neg B)$
(b) $(A \to B) \to [(A \vee C) \to (B \vee C)]$
(c) $A \to (B \to A)$
(d) $A \wedge B \leftrightarrow \neg B \vee \neg A$

[4] 3. In a certain country every inhabitant is either a truth teller (who always tells the truth) or a liar (who always lies). Traveling in this country you meet two of the inhabitants, Pat and Mel. Pat says, "If I am a truth teller, then Mel is a truth teller."

(a) Is Pat a truth teller or a liar?
(b) Is Mel a truth teller or a liar?

Provide mathematical justification for your answers.

[3] 4. Justify each step in the proof sequence of $P \wedge (Q \to R) \to [Q \to (P \wedge R)]$

1. P
2. $Q \to R$
3. Q
4. R
5. $P \wedge R$

[3] 5. Justify each step in the proof sequence of $\neg A \wedge B \wedge [B \to (A \vee C)] \to C$

1. $\neg A$
2. B
3. $B \to (A \vee C)$
4. $A \vee C$
5. $\neg(\neg A) \vee C$
6. $\neg A \to C$
7. C

[5] 6. Use propositional logic (not a truth table) to prove the validity of $\neg A \wedge (A \vee B) \to B$

[5] 7. Use propositional logic (not a truth table) to prove the validity of $(P \to Q) \wedge [P \to (Q \to R)] \to (P \to R)$

[5] 8. Use propositional logic (not a truth table) to prove the validity of $(P \to Q) \to (\neg Q \to \neg P)$