

The point values for each question appear within []. The total number of points for this assignment is 42.

[6] 1. Justify each step in the proof sequences.

(a) $P \wedge (Q \rightarrow R) \Rightarrow [Q \rightarrow (P \wedge R)]$

| Statement | Reason |
|----------------------|--------|
| 1. P | |
| 2. $Q \rightarrow R$ | |
| 3. Q | |
| 4. R | |
| 5. $P \wedge R$ | |

(b) $\neg A \wedge B \wedge [B \rightarrow (A \vee C)] \Rightarrow C$

| Statement | Reason |
|-------------------------------|--------|
| 1. $\neg A$ | |
| 2. B | |
| 3. $B \rightarrow (A \vee C)$ | |
| 4. $A \vee C$ | |
| 5. $\neg(\neg A) \vee C$ | |
| 6. $\neg A \rightarrow C$ | |
| 7. C | |

[20] 2. Provide a propositional logic proof sequence (not a truth table) to prove the validity of the following arguments.

(a) $\neg A \wedge (A \vee B) \Rightarrow B$

(b) $(P \rightarrow Q) \wedge [P \rightarrow (Q \rightarrow R)] \Rightarrow (P \rightarrow R)$

(c) $(P \rightarrow Q) \Rightarrow (\neg Q \rightarrow \neg P)$

(d) $A \rightarrow (B \rightarrow C) \Rightarrow B \rightarrow (A \rightarrow C)$

[16] 3. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. The domain is the whole world.

- $B(x)$ is “ x is a ball.”
- $R(x)$ is “ x is round.”
- $S(x)$ is “ x is used in soccer.”

- (a) All balls are round.
- (b) Some balls are not round.
- (c) Not all balls are soccer balls.
- (d) Some things are used in soccer.
- (e) All soccer balls are round.
- (f) Some balls are round, but soccer balls are not.
- (g) Every round ball is a soccer ball.
- (h) If a ball is not round, then it is not a soccer ball.