

The point values for each question appear within []. The total number of points for this assignment is 41.

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- [16] 1. Using the predicate symbols shown and appropriate quantifiers, write each English language statement as a predicate wff. The domain is the whole world.
- $B(x)$  is “ $x$  is a ball.”
  - $R(x)$  is “ $x$  is round.”
  - $S(x)$  is “ $x$  is used in soccer.”
- (a) All balls are round.  
(b) Some balls are not round.  
(c) Not all balls are soccer balls.  
(d) Some things are used in soccer.  
(e) All soccer balls are round.  
(f) Some balls are round, but soccer balls are not.  
(g) Every round ball is a soccer ball.  
(h) If a ball is not round, then it is not a soccer ball.
- [10] 2. Consider the following statement: The sum of two even integers is even.
- (a) Provide a direct proof.  
(b) Provide a proof by contradiction.
- [5] 3. Prove or disprove the following statement: For all integers  $n > 0$ :  $n^2 - n + 41$  is prime.
- [10] 4. Use mathematical induction to prove the following statements.
- (a) For every positive integer  $n$ :  $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$   
(b) For all integers  $n \geq 2$ :  $n! < n^n$