MATH 280 Discrete Mathematical Structures Assignment #8 Name ____________________

The point values for each question is given within []. The total number of points for this assignment is 13.

Consider the following recurrence relation defined over the set of positive integers:

$$T(1) = 1$$
$$T(n) = T(n-1) + n + 2$$

[2] 1. Write the first six elements of the sequence: $T(1)$, $T(2)$, $T(3)$, $T(4)$, $T(5)$, and $T(6)$.

[4] 2. Solve the recurrence relation, showing how you derived your solution. (Hint: When expanding the recurrences, keep the $+2$ terms on the end separate from the other constant terms (-1, -2, etc.) that appear in the middle during the expansion. Add up the $+2$ terms along the way, and the pattern for their $n-1$ case should be obvious. For the other constant terms (-1, -2, etc.), do not add them up along the way but rather see if you can collect them into a summation similar to the way shown in the handout [factor the negative sign outside of the summation]. The summation will be different from the one on the handout, but it will be one we have seen before. The pattern for the $n-1$ case then should be much clearer, allowing you to eliminate T altogether.)

Try out your solution on the values 1, 2, 3, 4, 5, and 6, and ensure that the results are the same as those you computed in Question 1.

[3] 3. Once you are convinced that your solution to Question 2 is correct, subject your solution to a more extensive test by writing a C++ or Python program that uses a recursive function modeling the original recurrence relation and a second function modeling your solution. Your program should demonstrate for a large range of values that the two functions produce the same results given equivalent inputs. You should be able to use test values much larger than those in the handout. Attach your source code and an abbreviated printout of the program's output.

[4] 4. Realizing that testing has its limitations, prove that your closed-form solution is correct using mathematical induction.