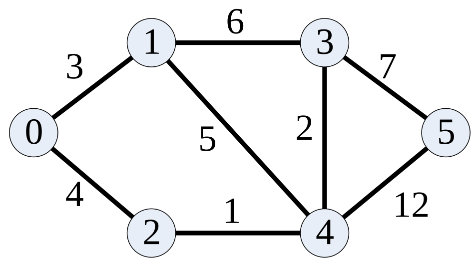
MATH 280 Discrete Mathematical Structures Assignment #9 Name ____________________

The point values for each question is given within []. The total number of points for this assignment is 35.

1. Consider graph G:



[2] (a) Does G contain an Eulerian path? Why or why not?

[2] (b) Is G Hamiltonian? Why or why not?

[4] (c) We can use Dijkstra's Algorithm to compute the shortest path from vertex 0 to all the other vertices in graph G. Complete each of the tables below that represent the state of the data structures used by Dijkstra's algorithm each time a vertex's shortest distance from vertex 0 becomes known.

Step 1

Vertex	Known	Distance	Previous
0	True	0	-1
1	False	3	0
2	False	4	0
3	False	∞	-1
4	False	∞	-1
5	False	∞	-1

Step 2

Vertex	Known	Distance	Previous
0	True	0	-1
1			
2			
3			
4			
5			

Step 3

Vertex	Known	Distance	Previous
0	True	0	-1
1			
2			
3			
4			
5			

Step 4

Vertex	Known	Distance	Previous
0	True	0	-1
1			
2			
3			
4			
5			

Step 5

Vertex	Known	Distance	Previous
0	True	0	-1
1			
2			
3			
4			
5			

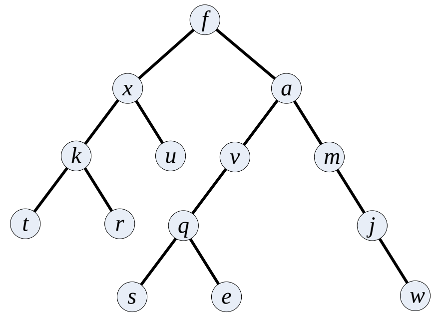
Step 6

Vertex	Known	Distance	Previous
0	True	0	-1
1			
2			
3			
4			
5			

[2] 2. Which of the graphs in Figure 9.2.11 of your textbook are isomorphic? Produce the bijection for one pair of isomorphic graphs.

[1] 3. How many edges does K_{10} have? Justify your answer.

[3] 4. Provide the pre-, in-, and postorder traversals for the following tree:



5. Consider the following table of letter frequencies for a particular data set:

Letter	Frequency	Code
A	15	
E	25	
I	10	
O	30	
U	15	
Y	5	

[3] (a) Construct a Huffman tree to be used to derive a minimal prefix code for the letters.

[2] (b) Complete the last column in the table with the bitstrings for the prefix code derived from your Huffman tree.

[3] 6. For each of the following mathematical structures circle G if the mathematical structure is a group, M if it is just a monoid, or N if it is neither a group nor a monoid.

(a) G M N $(\mathbb{R}, +)$

(b) G M N $(\mathbb{Z}, \cdot)$

(c) G M N $(\mathbb{N}, -)$

7. Determine which of the following mathematical structures are groups. For a group, you need to show closure, associativity, identity, and invertibility; otherwise, you need only show that one of these properties does not hold.

[3] (a) $(\{-1, 1\}, \cdot)$, where $\cdot$ is normal multiplication.

[3] (b) $(\mathbb{Z}, \diamond)$, where $a \diamond b$ is the larger of a and b.

[3] 8. Show that the set of even integers form a subgroup of $(\mathbb{Z}, +)$.

9. Consider the monoid $M_1 = (\mathbb{Z}, +)$, where $+$ is normal integer addition, and the monoid $M_2 = (A, +)$, where A is the set of 2×2 integer matrices and $+$ is normal matrix addition. Next consider the function $f : M_1 \to M_2$, such that $f(x) = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}$.

[3] (a) Show that f is a homomorphism from M_1 to M_2.

[1] (b) Is f an isomorphism?