MATH 280 Discrete Mathematical Structures Assignment #1

Name _____

The point values for each question is given within []. The total number of points for this assignment is 65.

[12] 1. Describe each of the following sets by listing its elements:

(a)
$$\{x \mid x \in \mathbb{N} \text{ and } x^2 - 5x + 6 = 0\}$$

(b) $\{\frac{1}{x} \mid x \in \{2, 4, 6, 8\}\}$
(c) $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 7\}$
(d) $\{x \mid x \in \mathbb{N} \text{ and } x^2 - 2x - 8 = 0\}$
(e) $\{x^2 \mid x \in \{-2, -1, 0, 1, 2\}\}$

(f) $\{x \in \mathbb{P} \mid x \text{ is a factor of } 24 \}$

$$A = \{2, 4, 5, 6, 8\}$$

$$B = \{1, 4, 5, 9\}$$

$$C = \{x \mid x \in \mathbb{Z} \text{ and } 2 \le x < 5\}$$

be subsets of $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Find

- (a) $A \cup B$
- (b) $A \cap B$
- (c) $A \cap C$
- (d) $B \cup C$
- (e) A B
- (f) A^C
- (g) $A \cap A^C$
- (h) $(A \cap B)^C$
- (i) C-B
- (j) $(C \cap B) \cup A^C$
- (k) $(B-A)^C \cap (A-B)$
- (1) $(C^C \cap B)^C$
- (m) *U^C*
- (n) $B \times C$
- (o) $\wp(B)$
- (p) $A \oplus B$
- (q) $B \oplus C$
- (r) |U|
- (s) $|U \times U|$
- (t) $|\wp(U \times U)|$
- (u) $|\emptyset|$
- (v) $|\{\varnothing\}|$

[8] 3. Let

 $A = \{x \mid x \text{ is a word that appears before } dog \text{ in an English language dictionary}\}$

 $B = \{x \mid x \text{ is a word that appears after } canary \text{ in an English language dictionary}\}$

 $C = \{x \mid x \text{ is a word of more than four characters}\}$

Which of the following are true statements?

- (a) $B \subseteq C$
- (b) $A \cup B = \{x \mid x \text{ is a word in an English language dictionary } \}$
- (c) $cat \in B \cap C^C$
- (d) $bamboo \in A B$
- [6] 4. Suppose *U* is an infinite universal set, and *A* and *B* are infinite subsets of *U*. Answer the following questions and justify your answer.
 - (a) Must A^C be infinite?
 - (b) Must $A \cup B$ be infinite?
 - (c) Must $A \cap B$ be infinite?
- [8] 5. Find the binary representation of each of the following positive integers:
 - (a) 64
 - (b) 67
 - (c) 28
 - (d) 256
- [10] 6. Calculate the following series:

(a)
$$\sum_{i=1}^{10} i$$

(b) $\sum_{i=1}^{5} 20$
(c) $\sum_{j=1}^{5} (2j+1)$
(d) $\sum_{i=-n}^{n} i$, for $n = 1, 2, 3, 4$
(e) $\prod_{i=1}^{5} i$

[2] 7. Draw the Venn diagram for $\bigcap_{i=1}^{4} A_i$