MATH 280 Discrete Mathematical Structures Assignment #3 Name ________________

The point values for each question is given within []. The total number of points for this assignment is 34.

Most of these problems have a single number for an answer. For full credit (or partial credit if your answer is incorrect), show how you obtained your result.

[4] 1. Given the truth values p **true** (or 1), q **false** (or 0), and r **true** (or 1), what is the truth value of each of the following statements?

(a) $p \wedge (q \vee r)$

(b) $(p \wedge q) \vee r$

(c) $\neg(p \vee q) \wedge r$

(d) $\neg p \vee (\neg q \wedge r)$

[8] 2. Complete the following truth tables. Show intermediate results in extra columns. Note any tautologies or contradictions.

p	q	$p \wedge (\neg p \vee \neg q)$
0	0	
0	1	
1	0	
1	1	

p	q	r	$(p \to q) \to [(p \vee r) \to (q \vee r)]$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

p	q	$p \to (q \to p)$
0	0	
0	1	
1	0	
1	1	

p	q	$p \wedge q \leftrightarrow \neg q \vee \neg p$
0	0	
0	1	
1	0	
1	1	

[5] 3. Consider the truth table for a new logical operator, $\diamond$:

P	Q	$P \diamond Q$
0	0	1
0	1	0
1	0	0
1	1	0

Prove that $\diamond$ is sufficient to represent *any* logical statement. To do this, you must show how to achieve $\neg$, $\wedge$, $\vee$, $\to$, and $\leftrightarrow$ using only $\diamond$. (Hint: Once you have shown how you can use $\diamond$ to implement a standard logical operator, you can use that standard operator to derive other standard operators.)

[3] 4. In a certain country every inhabitant is either a truth teller (who always tells the truth) or a liar (who always lies). Traveling in this country you meet two of the inhabitants, Pat and Mel. Pat says, “If I am a truth teller, then Mel is a truth teller.”

(a) Is Pat a truth teller or a liar?

(b) Is Mel a truth teller or a liar?

Provide mathematical justification for your answers.

[3] 5. Justify each step in the proof sequence of $P \wedge (Q \rightarrow R) \Rightarrow [Q \rightarrow (P \wedge R)]$

1. P
2. $Q \rightarrow R$
3. Q
4. R
5. $P \wedge R$

[3] 6. Justify each step in the proof sequence of $\neg A \wedge B \wedge [B \rightarrow (A \vee C)] \Rightarrow C$

1. $\neg A$
2. B
3. $B \rightarrow (A \vee C)$
4. $A \vee C$
5. $\neg(\neg A) \vee C$
6. $\neg A \rightarrow C$
7. C

[4] 7. Use propositional logic (not a truth table) to prove the validity of $\neg A \wedge (A \vee B) \Rightarrow B$

[4] 8. Use propositional logic (not a truth table) to prove the validity of $(P \rightarrow Q) \wedge [P \rightarrow (Q \rightarrow R)] \Rightarrow (P \rightarrow R)$