

The point values for each question is given within []. The total number of points for this assignment is 45.

- [20] 1. For each of the following relations on the set $\{1, 2, 3, 4\}$, check **R** if the relation is reflexive, check **S** if the relation is symmetric, check **A** if the relation is antisymmetric, and check **T** if the relation is transitive.

(a) $\{(1, 2), (2, 3), (3, 4)\}$

R	S	A	T

(b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

R	S	A	T

(c) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

R	S	A	T

(d) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

R	S	A	T

(e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

R	S	A	T

- [16] 2. For each of the following relations on the set of integers, \mathbb{Z} , check **R** if the relation is reflexive, check **S** if the relation is symmetric, check **A** if the relation is antisymmetric, and check **T** if the relation is transitive.

(a) $x R y \leftrightarrow xy \geq 1$

R	S	A	T

(b) $x R y \leftrightarrow \exists k \in \mathbb{Z}$ such that $x = ky$

R	S	A	T

(c) $x R y \leftrightarrow x = y + 1$ or $x = y - 1$

R	S	A	T

(d) $x R y \leftrightarrow x \geq y^2$

R	S	A	T

- [3] 3. Draw a Hasse diagram for each of the following partially ordered sets.

(a) $S = \{2, 4, 5, 10, 12, 20, 25\}$, $x R y \leftrightarrow x$ divides y (said another way, x is a factor of y)

(b) $T = \wp(\{1, 2, 3\})$, for $A, B \in T$, $A R B \leftrightarrow A \subseteq B$

- [2] 4. Find the maximal and minimal elements of

(a) the poset described in Question 3a

(b) the poset described in Question 3b

- [4] 5. In the C++ programming language, character strings can be represented by objects of type `std::string`. If s is a `std::string` object, $s.length()$ is the number of characters that make up s . All strings in C++ have a finite length. For all integers $0 \leq i < s.length()$, the expression $s[i]$ evaluates to the character at position i in string s . As in all C-derived languages, the first character in string s is $s[0]$.

(a) Let R be the relation on the set of all C++ strings such that $s R t \leftrightarrow s.length() = t.length()$. Show that R is an equivalence relation on the set of all C++ strings.

(b) Let R be the relation on the set of all non-empty C++ strings such that $s R t \leftrightarrow s$ is a prefix of t . Show that R is a partial order on the set of C++ strings.