MATH 280 Discrete Mathematical Structures Assignment #7 Name ____________________

The point values for each question is given within []. The total number of points for this assignment is 25.

[2] 1. Given set $A = \{0,1,2,3\}$ and relation $R = \{(0,1),(0,2),(1,1),(1,3),(2,2),(3,0)\}$, compute R^+.

[4] 2. Let $A = \{1,2,3,4\}$ and $B = \{a,b,c,d\}$. Determine which of the following relations are functions.

(a) $f \subseteq A \times B$, where $f = \{(1,a),(2,b),(3,c),(4,d)\}$.

(b) $g \subseteq A \times B$, where $g = \{(1,a),(2,a),(3,b),(4,d)\}$.

(c) $h \subseteq A \times B$, where $h = \{(1,a),(2,b),(3,c)\}$.

(d) $k \subseteq A \times B$, where $k = \{(1,a),(2,b),(2,c),(3,a),(4,a)\}$.

[10] 3. For each of the following mappings from the given domain to the given codomain, write **N** if the mapping is **not** a function, write **F** if the mapping is a function that is neither injective nor surjective, write **S** if the mapping is a surjective function, write **I** if the mapping is an injective function, write **B** if the mapping is a bijective function, and, if the mapping is a bijection, provide f^{-1}.

(a) $f : \mathbb{Z} \to \mathbb{N}, \quad f(x) = x^2 + 1$

(b) $f : \mathbb{Z} \to \mathbb{Q}, \quad f(x) = \frac{1}{x}$

(c) $f : \mathbb{Z} \times \mathbb{N} \to \mathbb{Q}, \quad f(x,y) = \frac{x}{y+1}$

(d) $f : \mathbb{N} \to \mathbb{N}, \quad f(x) = 2^x$

(e) $f : \mathbb{N} \to \mathbb{N}, \quad f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

[2] 4. If A and B are both finite sets, how many different functions are there from A to B? Justify your answer. (Note: This is a combinatorics problem. As usual, once you think you have found the solution, see if your solution works for a couple of small sets. If it does not work, you know your solution is wrong; if it does work, you have some evidence that your solution may be correct. Your "testing" of your solution in this manner is not the required justification; justify your answer with concepts from combinatorics.)

[7] 5. Consider the following recurrence relation:

$$T(1) = 1$$
$$T(n) = 2T(n-1) + 1$$

(a) Write the first five elements of the sequence.

(b) Solve the recurrence relation subject to the basis step.

(c) Test your solution to 5b by writing a recursive function modeling the original recurrence relation and a second function modeling your solution. Ensure that the two functions produce the same results given equivalent inputs.

(d) Realizing that testing has its limitations, prove that your closed-form solution is correct.