MATH 280 Discrete Mathematical Structures Assignment #12   Name _____

The point values for each question is given within []. The total number of points for this assignment is 40.

[2]   1. Consider the monoid $(\mathbb{Z}, +)$. Compute $\langle\{5\}\rangle$, the submonoid generated by the set $\{5\}$.

[4]   2. Consider the grammar $\langle T, N, S, P \rangle$, where $T = \{+, *, (,), n\}$, $N = \{S\}$, $S = S$, and P is defined by

$$S \quad \to \quad S + S \mid S * S \mid (S) \mid n$$

Provide parse trees for the following strings:

   (a) $n * (n + n)$
   (b) $n * n + n$

[4]   3. Consider the grammar $\langle T, N, S, P \rangle$, where $T = \{+, *, (,), n\}$, $N = \{E, T, F\}$, $S = E$, and P is defined by
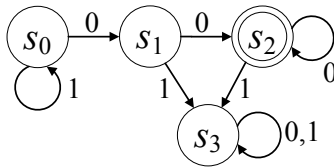
$$\begin{aligned} E &\to E + T \mid T \\ T &\to T * F \mid F \\ F &\to (E) \mid n \end{aligned}$$

Provide parse trees for the following strings:

   (a) $n * (n + n)$
   (b) $n * n + n$

[5]   4. Consider the set of all bitstrings that begin and end with a 1.

   (a) Provide a grammar for the language.
   (b) Use your grammar to produce a parse tree for the string 10101

[3]   5. Give a regular expression for the set recognized by the following finite automaton:

[6]   6. Give a regular expression for set of bitstrings

   (a) containing an even number of 0s
   (b) that begin with a 0 and end with 010

[3]   7. Provide a finite automaton that recognizes the set of bitstrings that begin with a 0 and end with 010.

[5]   8. Find a context-free grammar that generates the language $L = \{ss^R \mid s \in \{0,1\}^* \text{ and } s^R \text{ is the reverse of string } s\}$.

[8]   9. Consider the regular expresson $(1^*0(01)^*)\mid(00^*)$.

   (a) Provide a context-free grammar $G = \langle T, N, S, P \rangle$ for strings represented by the regular expression.
   (b) Given your G, draw a parse tree for 00000000.
   (c) Given your G, draw a parse tree for 00101.
   (d) Given your G, draw a parse tree for 1110.