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The point values for each question is given within []. The total number of points for this assignment is 10 .
Please submit your answers electronically to eclass. You may typeset your solutions with $\mathrm{ET}_{\mathrm{E}} \mathrm{X}$ or use Microsoft Word's equation editor.
[2] 1. Prove using mathematical induction that the sum of the first $n$ even integers is $n^{2}+n$.
[3] 2. A particular algorithm can solve a problem of input size 100 in 5 milliseconds. Estimate the size of the problem the algorithm can solve in 1 minute if the algorithm's asymptotic complexity is each of the following:
(a) $\Theta(n)$
(b) $\Theta(\log n)$
(c) $\Theta(n \log n)$
(d) $\Theta\left(n^{2}\right)$
(e) $\Theta\left(n^{3}\right)$
(f) $\Theta\left(2^{n}\right)$

Provide mathematical justification for your answers.
[5] 3. Determine the $\Theta$ asymptotic time complexity of each of the following $\mathrm{C}++$ code fragments ( n is the data size):
(a)

```
int sum = 0;
for ( int i = 0; i < n; i++ )
    sum++;
```

(b) int sum $=0$;
for ( int $i=0 ; i<n$; i++ )
for ( int $j=0 ; j<n$; $j++$ )
sum++;
(c)

```
int sum = (n < 100000)? n : 100000;
```

(d) int sum $=0$;
for ( int $i=0 ; i<n$; $i++$ )
for ( int $j=0 ; j<n$; j++ )
for ( int $k=0 ; k<n ; k++$ )
sum++;
(e) int sum $=0$;

```
for ( int i = 0; i < n; i++ )
    sum++;
for ( int i = 0; i < 2*n; i++ )
    sum++;
```

(f)

```
int sum = 0;
for ( int i = 0; i < n; i++ )
    for ( int j = 0; j < n*n; j++ )
        sum++;
```

(g) int sum = 0;
for ( int $i=0 ; i<100 ; i++$ )
sum++;
(h) int sum = 0;
for ( int $i=0 ; i<n ; i++$ )
for (int $j=0 ; j<i ; j++$ )
sum++;
(i) int sum $=0$;
for ( int $i=0$; $i<n$; $i++$ )
for ( int $j=0 ; j<i * i ; j++$ )
for ( int $k=0 ; k<j ; k++$ )
sum++;
(j)

```
int sum = 0;
for ( int i = n; i > 0; i /= 2 )
    sum++;
```

